On the polarization properties of the charmed baryon Λ^+_c in the $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ decay

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Abstract

The polarization properties of the charmed Λ^+_c baryon are investigated in weak non–leptonic four–body $\Lambda^+_c \to p+K^-+\pi^++\pi^0$ decay. The probability of this decay and the angular distribution of the probability are calculated in the effective quark model with chiral $U(3)\times U(3)$ symmetry incorporating Heavy Quark Effective theory (HQET) and the extended Nambu–Jona–Lasinio model with a linear realization of chiral $U(3) \times U(3)$ symmetry. The theoretical value of the probability of the decay $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ relative to the probability of the decay $\Lambda^+_c \to p + K^- + \pi^+$ does not contain free parameters and fits well experimental data. The application of the obtained results to the analysis of the polarization of the Λ^+_c produced in the processes of photo and hadroproduction is discussed.

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1 Introduction

It is known that in reactions of photo and hadroproduction the charmed baryon Λ^+_c is produced polarized [1]. The analysis of the Λ^+_c polarization via the investigation of the decay products should give an understanding of the mechanism of the charmed baryon production at high energies.

Recently [2] we have given a theoretical analysis of the polarization properties of the Λ^+_c in the mode $\Lambda^+_c \to p + K^- + \pi^+$. This is the most favourable mode of the Λ^+_c decays from the experimental point of view. From the theoretical point of view this mode is the most difficult case of the analysis of the weak non–leptonic decays of the Λ^+_c baryon [1,2]. Indeed, for the calculation of the matrix element of the transition $\Lambda^+_c \to p + K^- + \pi^+$ the baryonic and mesonic degrees of freedom cannot be fully factorized.

In spite of these theoretical difficulties the problem of the theoretical analysis of the decay $\Lambda^+_c \to p + K^- + \pi^+$ has been successfully solved within the effective quark model with chiral $U(3) \times U(3)$ symmetry incorporating Heavy Quark Effective Theory (HQET) [3,4] and the extended Nambu–Jona–Lasinio (ENJL) model with a linear realization of chiral $U(3) \times U(3)$ symmetry [5–7] [1]. Such an effective quark model with chiral $U(3) \times U(3)$ symmetry motivated by the low–energy effective QCD with a linearly rising interquark potential responsible for a quark confinement [9] describes well low–energy properties of light and heavy mesons [5,6] as well as the octet and decuplet of light baryons [7].

In the effective quark model with chiral $U(3) \times U(3)$ symmetry (i) baryons are the three–quark states [10] and do not contain any bound diquark states, then (ii) the spinorial structure of the three–quark currents is defined as the products of the axial–vector diquark densities $[\bar{q}^c{}_i(x)\gamma^\mu q_j(x)]$ and a quark field $q_k(x)$ transforming under $SU(3)_f \times SU(3)_c$ group like $(\underline{6}_f, \tilde{\underline{3}}_c)$ and $(\underline{3}_f, \underline{3}_c)$ multiplets, respectively, where i, j and k are the colour indices running through $i = 1, 2, 3$ and $q = u, d$ or s quark field. This agrees with the structure of the three–quark currents used for the investigation of the properties of baryons within QCD sum rules approach [11]. As has been shown in Ref.[9] this is caused by the dynamics of strong low–energy interactions imposed by a linearly rising interquark potential. The fixed structure of the three–quark currents allows to describe all variety of low–energy interactions of baryon octet and decuplet in terms of the phenomenological coupling constant $g_{\rm B}$. The coupling constants $g_{\pi\rm NN}$, $g_{\pi\rm N\Delta}$ and $g_{\gamma\rm N\Delta}$ interactions, and the $\sigma_{\pi\rm N}$–term of the low–energy πN–scattering have been calculated in good agreement with the experimental data and other phenomenological approaches based on QCD [7,12].

In this paper we apply the effective quark model with chiral $U(3) \times U(3)$ symmetry [2,5–7] to the investigation of the polarization properties of of the Λ^+_c baryon in weak non–leptonic four–body decays and treat the most favourable experimentally four–body mode $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$. The experimental value of the probability of this decay is equal to [13]

$$B(\Lambda^+_c \to pK^-\pi^+\pi^0)_{\rm exp} = (3.4 \pm 1.0)\,\%. \tag{1.1}$$

Relative to the decay $\Lambda^+_c \to \mathrm{p} + \mathrm{K}^- + \pi^+$ the experimental probability of which is $B(\Lambda^+_c \to pK^-\pi^+) = 0.050 \pm 0.013$ [13] the probability of the decay $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$

[1]All results obtained below are valid for the Linear Sigma Model (LσM) [8] supplemented by HQET as well.

reads

$$B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{\rm exp} = (0.68 \pm 0.27). \tag{1.2}$$

We would like to emphasize that the weak non–leptonic four–body mode $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ as well as the mode $\Lambda_c^+ \to p + K^- + \pi^+$ is rather difficult for the theoretical analysis [1,2], since baryonic and mesonic degrees of freedom cannot be fully factorized.

For the theoretical analysis of the weak non–leptonic decays of the Λ_c^+ baryon we would use the effective low–energy Lagrangian [2] (see also Refs.[12,14])

$$\begin{aligned}\mathcal{L}_{\rm eff}(x) &= -\frac{G_F}{\sqrt{2}}\,V^*_{cs}\,V_{ud}\Big\{C_1(\Lambda_\chi)\,[\bar{s}(x)\,\gamma^\mu(1-\gamma^5)\,c(x)]\,[\bar{u}(x)\,\gamma_\mu(1-\gamma^5)\,d(x)]\\ &\qquad + C_2(\Lambda_\chi)\,[\bar{u}(x)\,\gamma^\mu(1-\gamma^5)\,c(x)]\,[\bar{s}(x)\,\gamma_\mu(1-\gamma^5)\,d(x)]\Big\},\end{aligned} \tag{1.3}$$

where $G_F = 1.166\times 10^{-5}\ {\rm GeV}^{-2}$ is the Fermi weak constant, V^*_{cs} and V_{ud} are the elements of the CKM–mixing matrix, $C_i(\Lambda_\chi)\,(i = 1,2)$ are the Wilson coefficients caused by the strong quark–gluon interactions at scales $p > \Lambda_\chi$ (short–distance contributions), where $\Lambda_\chi = 940\,{\rm MeV}$ is the scale of spontaneous breaking of chiral symmetry (SBχS) [2,5–7]. The numerical values of the coefficients $C_1(\Lambda_\chi) = 1.24$ and $C_2(\Lambda_\chi) = -0.47$ have been calculated in Ref.[2].

Following Ref.[2] for the calculation of the probability of the decay $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ we suggest to use the effective Lagrangian Eq.(1.3) reduced to the form

$$\mathcal{L}_{\rm eff}(x) = -\frac{G_F}{\sqrt{2}}\,V^*_{cs}\,V_{ud}\,\bar{C}_1(\Lambda_\chi)\,[\bar{s}(x)\,\gamma_\mu(1-\gamma^5)\,c(x)]\,[\bar{u}(x)\,\gamma^\mu(1-\gamma^5)\,d(x)] \tag{1.4}$$

by means of a Fierz transformation [2], where $\bar{C}_1(\Lambda_\chi) = C_1(\Lambda_\chi) + C_2(\Lambda_\chi)/N$ with $N = 3$, the number of quark colour degrees of freedom[2].

The paper is organized as follows. In Sect. 2 we calculate the amplitude of the decay mode $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$. In Sect. 3 we calculate the angular distribution of the probability and the probability of the decay $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ relative to the probability of the $\Lambda_c^+ \to p + K^- + \pi^+$. In Sect. 4 we analyse the polarization properties of the charmed baryon Λ_c^+. In the Conclusion we discuss the obtained results.

2 Amplitude of the $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ decay

The amplitude of the decay $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ decay we define in the usual way [2,12]

$$\frac{\mathcal{M}(\Lambda_c^+(Q) \to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3))}{\sqrt{2E_{\Lambda_c^+}V\,2E_pV\,2E_{K^-}V\,2E_{\pi^+}V\,2E_{\pi^0}V}} = \langle p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)|\mathcal{L}_{\rm eff}(0)|\Lambda_c^+(Q)\rangle, \tag{2.1}$$

where $E_i\,(i = \Lambda_c^+, p, K^-, \pi^+, \pi^0)$ are the energies of the Λ_c^+, the proton and mesons, respectively.

[2] We would like to accentuate that our approach to non–leptonic decays of charmed baryons agrees in principle with the current–algebra analysis of non–leptonic decays of light and charmed baryons based on $(V - A)\times(V - A)$ effective coupling developed by Scadron *et al.* in Refs.[15].

Since experimentally the probability of the decay mode $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ is measured relative to the probability of the $\Lambda^+_c \to p + K^- + \pi^+$ decay, so that we would treat it with respect to the probability of the decay $\Lambda^+_c \to p + K^- + \pi^+$ the partial width of which has been calculated in Ref.[2] and reads

$$\Gamma(\Lambda^+_c \to p\,K^-\pi^+) = |G_F\,V^*_{cs}\,V_{ud}\,\bar{C}_1(\Lambda_\chi)|^2\left[g_{\pi\mathrm{NN}}\,\frac{4}{5}\,\frac{g_{\mathrm{C}}}{g_{\mathrm{B}}}\,\frac{F_\pi\Lambda_\chi}{m^2}\right]^2 \times \left[\frac{5M^5_{\Lambda^+_c}}{512\pi^3}\right] \times f(\xi). \qquad (2.2)$$

The function $f(\xi)$ is determined by the integral [2]

$$f(\xi) = \int\limits_{\xi}^{1+\xi^2/4}\left(1 - \frac{3}{5}\,x + \frac{2}{15}\,x^2 + \frac{7}{60}\,\xi^2 - \frac{2}{5}\,\frac{\xi^2}{x}\right)x\,\sqrt{x^2-\xi^2}\,dx = 0.065, \qquad (2.3)$$

where $\xi = 2M_p/M_{\Lambda^+_c}$. The numerical value has been obtained at $M_{\Lambda^+_c} = 2285\,\mathrm{MeV}$ and $M_p = 938\,\mathrm{MeV}$, the mass of the Λ^+_c baryon and the proton, respectively, and in the chiral limit, i.e. at zero masses of daugther mesons. The coupling constants g_{B} and g_{C} determine the interactions of the proton and the Λ^+_c baryon with the three–quark currents $\eta_{\mathrm{N}}(x) = -\varepsilon^{ijk}[\bar{u^c}_i(x)\gamma^\mu u_j(x)]\gamma_\mu\gamma^5 d_k(x)$ and $\bar{\eta}_{\Lambda^+_c}(x) = \varepsilon^{ijk}\bar{c}_i(x)\gamma_\mu\gamma^5[\bar{d}_j(x)\gamma^\mu u^c_k(x)]$, respectively [2,7]:

$$\mathcal{L}_{\mathrm{int}}(x) = \frac{g_{\mathrm{B}}}{\sqrt{2}}\,\bar{\psi}_p(x)\,\eta_{\mathrm{N}}(x) + \frac{g_{\mathrm{C}}}{\sqrt{2}}\,\bar{\eta}_{\Lambda^+_c}(x)\,\psi_{\Lambda^+_c}(x) + \mathrm{h.c.}. \qquad (2.4)$$

Here $\psi_p(x)$ and $\psi_{\Lambda^+_c}(x)$ are the interpolating fields of the proton and the Λ^+_c baryon. The coupling constant g_{B} has been related in Ref.[7] to the quark condensate $\langle\bar{q}(0)q(0)\rangle = -\,(255\,\mathrm{MeV})^3$, the constituent quark mass $m = 330\,\mathrm{MeV}$ calculated in the chiral limit[3], the leptonic coupling constant $F_\pi = 92.4\,\mathrm{MeV}$ of pions calculated in the chiral limit, the πNN coupling constant $g_{\pi\mathrm{NN}} = 13.4$ and as well as the mass of the proton M_{p}:

$$g_{\pi\mathrm{NN}} = g^2_{\mathrm{B}}\,\frac{2m}{3F_\pi}\,\frac{\langle\bar{q}(0)q(0)\rangle^2}{M^2_p}. \qquad (2.5)$$

Numerically g_{B} is equal to $g_{\mathrm{B}} = 1.34\times 10^{-4}\,\mathrm{MeV}$ [7]. The coupling constant g_{C} has been fixed in Ref.[2] through the experimental value of the partial width of the decay $\Lambda^+_c \to p + K^- + \pi^+$. The coupling constant g_{C} appears in all partial widths of the decay modes of the Λ^+_c baryon and cancels itself in the ratio

$$B(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+) = \frac{\Gamma(\Lambda^+_c \to pK^-\pi^+\pi^0)}{\Gamma(\Lambda^+_c \to p\,K^-\pi^+)}. \qquad (2.6)$$

The amplitude of the decay $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ we calculate in the tree–meson approximation and in the chiral limit [2]

$$\frac{\mathcal{M}(\Lambda^+_c(Q) \to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3))}{\sqrt{2E_{\Lambda^+_c}V\,2E_pV\,2E_{K^-}V\,2E_{\pi^+}V\,2E_{\pi^0}V}} = \langle p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)|\mathcal{L}_{\mathrm{eff}}(0)|\Lambda^+_c(Q)\rangle =$$
$$= -\frac{G_F}{\sqrt{2}}\,V^*_{cs}\,V_{ud}\,\bar{C}_1(\Lambda_\chi)\,\langle p(q)K^-(q_1)|\bar{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda^+_c(Q)\rangle$$
$$\times\langle\pi^+(q_2)\pi^0(q_3)|\bar{u}(0)\,\gamma^\mu(1-\gamma^5)\,d(0)|0\rangle, \qquad (2.7)$$

[3]This agrees with the results obtained by Elias and Scadron [16].

The matrix element of the transition $\Lambda^+_c \to p + K^-$ has been calculated in Ref.[2] and reads

$$\begin{aligned}
&\sqrt{2E_{\Lambda^+_c}V\,2E_pV\,2E_{K^-}V}\,\langle p(q)K^-(q_-)|\bar{s}(0)\,\gamma_\mu(1-\gamma^5)\,c(0)|\Lambda^+_c(Q)\rangle = \\
&= ig_{\pi\mathrm{NN}}\,\frac{4}{5}\,\frac{g_{\mathrm{C}}}{g_{\mathrm{B}}}\,\frac{\Lambda_\chi}{m^2}\,\bar{u}_p(q,\sigma'\,)\,[2\,v_\mu(1-\gamma^5)+\gamma_\mu(1+\gamma^5)]\,u_{\Lambda^+_c}(Q,\sigma) = \\
&= ig_{\pi\mathrm{NN}}\,\frac{4}{5}\,\frac{g_{\mathrm{C}}}{g_{\mathrm{B}}}\,\frac{\Lambda_\chi}{m^2}\,\bar{u}_p(q,\sigma'\,)\,(1-\gamma^5)\,(2\,v_\mu+\gamma_\mu)\,u_{\Lambda^+_c}(Q,\sigma),
\end{aligned}\tag{2.8}$$

where $\bar{u}_p(q,\sigma'\,)$ and $u_{\Lambda^+_c}(Q,\sigma)$ are the Dirac bispinors of the proton and the Λ^+_c baryon, v^μ is a 4–velocity of the Λ^+_c baryon defined by $Q^\mu = M_{\Lambda^+_c}\,v^\mu$.

The matrix element of the transition $0 \to \pi^+ + \pi^0$ has been calculated in [5] and reads

$$\sqrt{2E_{\pi^+}V\,2E_{\pi^0}V}\langle\pi^+(q_2)\pi^0(q_3)|\bar{u}(0)\gamma^\mu(1-\gamma^5)\,d(0)|0\rangle = -\sqrt{2}\,(q_2-q_3)^\mu. \tag{2.9}$$

Hence, the amplitude of the decay $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ is given by

$$\begin{aligned}
&\mathcal{M}(\Lambda^+_c(Q) \to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)) = i\,G_F\,V^*_{cs}\,V_{ud}\,\bar{C}_1(\Lambda_\chi) \\
&\times\frac{4}{5}\,\frac{g_{\pi\mathrm{NN}}}{M_{\Lambda^+_c}}\left[\frac{g_{\mathrm{C}}}{g_{\mathrm{B}}}\,\frac{\Lambda_\chi}{m^2}\right]\bar{u}_p(q,\sigma'\,)\,(1-\gamma^5)\,[2Q\cdot(q_2-q_3)+M_{\Lambda^+_c}(\hat{q}_2-\hat{q}_3)]\,u_{\Lambda^+_c}(Q,\sigma).
\end{aligned}\tag{2.10}$$

Now we can proceed to the evaluation of the probability of the $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ decay.

3 Probability and angular distribution of the decay $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$

The differential partial width of the $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ decay is determined by

$$\begin{aligned}
&d\Gamma(\Lambda^+_c \to pK^-\pi^+\pi^0) = \frac{1}{2M_{\Lambda^+_c}}\,\overline{|\mathcal{M}(\Lambda^+_c(Q) \to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3))|^2} \\
&\times(2\pi)^4\,\delta^{(4)}(Q-q-q_1-q_2-q_3)\,\frac{d^3q}{(2\pi)^3 2E_p}\,\frac{d^3q_1}{(2\pi)^3 2E_{K^-}}\,\frac{d^3q_2}{(2\pi)^3 2E_{\pi^+}}\,\frac{d^3q_1}{(2\pi)^3 2E_{\pi^0}}.
\end{aligned}\tag{3.1}$$

We calculate the quantity $\overline{|\mathcal{M}(\Lambda^+_c(Q) \to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3))|^2}$ for the polarized Λ^+_c and unpolarized proton

$$\begin{aligned}
&\overline{|\mathcal{M}(\Lambda^+_c(Q) \to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3))|^2} = |G_F\,V^*_{cs}\,V_{ud}\,\bar{C}_1(\Lambda_\chi)|^2\,\left[\frac{4}{5}\,\frac{g_{\pi\mathrm{NN}}}{M_{\Lambda^+_c}}\,\frac{g_{\mathrm{C}}}{g_{\mathrm{B}}}\,\frac{\Lambda_\chi}{m^2}\right]^2 \\
&\times\frac{1}{2}\,\mathrm{tr}\{(M_{\Lambda^+_c}+\hat{Q})(1+\gamma^5\hat{\omega}_{\Lambda^+_c})[2Q\cdot(q_2-q_3)+M_{\Lambda^+_c}(\hat{q}_2-\hat{q}_3)](1+\gamma^5)(M_p+\hat{q})(1-\gamma^5) \\
&\times[2Q\cdot(q_2-q_3)+M_{\Lambda^+_c}(\hat{q}_2-\hat{q}_3)]\},
\end{aligned}\tag{3.2}$$

where $\omega^\mu_{\Lambda^+_c}$ is a space–like unit vector, $\omega^2_{\Lambda^+_c} = -1$, orthogonal to the 4–momentum of the Λ^+_c, $Q\cdot\omega_{\Lambda^+_c} = 0$. It is related to the direction of the Λ^+_c spin defined by

$$\omega^\mu_{\Lambda^+_c} = \left(\frac{\vec{Q}\cdot\vec{\omega}_{\Lambda^+_c}}{M_{\Lambda^+_c}},\vec{\omega}_{\Lambda^+_c}+\frac{\vec{Q}(\vec{Q}\cdot\vec{\omega}_{\Lambda^+_c})}{M_{\Lambda^+_c}(E_{\Lambda^+_c}+M_{\Lambda^+_c})}\right), \tag{3.3}$$

where $\vec{\omega}^{\,2}_{\Lambda^+_c} = 1$. At the rest frame of the Λ^+_c we have $\omega^{\mu}_{\Lambda^+_c} = (0, \vec{\omega}_{\Lambda^+_c})$.

For the differential branching ratio $B(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+)$ defined by Eq.(2.6) we get

$$dB(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+) = \frac{1024\pi^3}{1.3 M^8_{\Lambda^+_c}}\,\frac{1}{F^2_{\pi}}\,\frac{1}{2}\,\mathrm{tr}\{(M_{\Lambda^+_c} + \hat{Q})(1 + \gamma^5\hat{\omega}_{\Lambda^+_c})$$
$$\times [2Q\cdot(q_2 - q_3) + M_{\Lambda^+_c}(\hat{q}_2 - \hat{q}_3)](1+\gamma^5)(M_p + \hat{q})(1-\gamma^5)$$
$$\times [2Q\cdot(q_2 - q_3) + M_{\Lambda^+_c}(\hat{q}_2 - \hat{q}_3)]\}\,(2\pi)^4\,\delta^{(4)}(Q - q - q_1 - q_2 - q_3)$$
$$\times \frac{d^3q}{(2\pi)^3 2E_p}\,\frac{d^3q_1}{(2\pi)^3 2E_{K^-}}\,\frac{d^3q_2}{(2\pi)^3 2E_{\pi^+}}\,\frac{d^3q_3}{(2\pi)^3 2E_{\pi^0}}. \tag{3.4}$$

The trace amounts to

$$\begin{aligned}\frac{1}{2}\,\mathrm{tr}\{\ldots\} &= 16\,Q\cdot q\,(Q\cdot(q_2 - q_3))^2\\ &+ M_{\Lambda^+_c}[16\,Q\cdot q\,Q\cdot(q_2 - q_3)\,(q_2 - q_3)\cdot\omega_{\Lambda^+_c} - 32\,(Q\cdot(q_2 - q_3))^2\,q\cdot\omega_{\Lambda^+_c}]\\ &+ M^2_{\Lambda^+_c}[24\,Q\cdot(q_2 - q_3)\,q\cdot(q_2 - q_3) - 4\,Q\cdot q\,(q_2 - q_3)^2]\\ &+ M^3_{\Lambda^+_c}[8\,q\cdot(q_2 - q_3)\,(q_2 - q_3)\cdot\omega_{\Lambda^+_c} - 4\,(q_2 - q_3)^2\,q\cdot\omega_{\Lambda^+_c}].\end{aligned} \tag{3.5}$$

For the integration over the momenta of π mesons it is useful to apply the formula [2]

$$\int (q_2 - q_3)_{\alpha}(q_2 - q_3)_{\beta}\,\delta^{(4)}(P - q_2 - q_3)\frac{d^3q_2}{2E_{\pi^+}}\frac{d^3q_3}{2E_{\pi^0}} = \frac{\pi}{6}\Big(-P^2\,g_{\alpha\beta} + P_{\alpha}P_{\beta}\Big), \tag{3.6}$$

where $P = Q - q - q_1$. Integrating over the momenta of pions we arrive at the following expression for the differential branching ratio $B(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+)$:

$$dB(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+) = \frac{2.1}{4\pi^4}\,\frac{1}{M^8_{\Lambda^+_c}}\,\frac{1}{F^2_{\pi}}\,\{4\,Q\cdot q\,((Q\cdot P)^2 - Q^2P^2)$$
$$+M_{\Lambda^+_c}\,[4\,Q\cdot q\,Q\cdot P\,P\cdot\omega_{\Lambda^+_c} - 8\,((Q\cdot P)^2 - Q^2P^2)\,P\cdot\omega_{\Lambda^+_c}] + M^2_{\Lambda^+_c}\,(-3\,Q\cdot q\,P^2$$
$$+6\,Q\cdot P\,q\cdot P) + M^3_{\Lambda^+_c}\,(P^2\,q\cdot\omega_{\Lambda^+_c} + 2\,q\cdot P\,P\cdot\omega_{\Lambda^+_c})\}\,\frac{d^3q}{E_p}\,\frac{d^3q_1}{E_{K^-}}. \tag{3.7}$$

After the integration over the momenta of the K^- meson and the energies of the proton we obtain the angular distribution of the probability of the decay mode $\Lambda^+_c \to p + K^- + \pi^+ + \pi^0$ relative to the probability of the decay $\Lambda^+_c \to p + K^- + \pi^+$ in the rest frame of the Λ^+_c baryon:

$$4\pi\,\frac{dB}{d\Omega_{\vec{n}_p}}(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+) = 0.87\,(1 - 0.09\,\vec{n}_p\cdot\vec{\omega}_{\Lambda^+_c}), \tag{3.8}$$

where $\vec{n}_p = \vec{q}/|\vec{q}\,|$ is a unit vector directed along the momentum of the proton and $\Omega_{\vec{n}_p}$ is the solid angle of the unit vector $\vec{n}_p$.

Integrating the angular distribution Eq.(3.8) over the solid angle $\Omega_{\vec{n}_p}$ we obtain the total branching ratio

$$B(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+) = 0.87. \tag{3.9}$$

The theoretical value fits well the experimental data Eq.(1.2): $B(\Lambda^+_c \to pK^-\pi^+\pi^0/\Lambda^+_c \to pK^-\pi^+)_{\rm exp} = (0.68 \pm 0.27)$.

4 Polarization of the charmed baryon Λ_c^+

The formula Eq.(3.8) describes the polarization of the charmed Λ_c^+ baryon relative to the momentum of the proton in the decay mode $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$. If the spin of the Λ_c^+ is parallel to the momentum of the proton, the right–handed (R) polarization, the scalar product $\vec{\omega}_{\Lambda_c^+} \cdot \vec{n}_p$ amounts to $\vec{\omega}_{\Lambda_c^+} \cdot \vec{n}_p = \cos\vartheta$. The angular distribution of the probability reads

$$4\pi \frac{dB}{d\Omega_{\vec{n}_p}}(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{(\mathrm{R})} = 0.87\,(1 - 0.09\,\cos\vartheta). \tag{4.1}$$

In turn, for the left–handed (L) polarization of the Λ_c^+, the spin of the Λ_c^+ is anti–parallel to the momentum of the proton, the scalar product reads $(\vec{\omega}_{\Lambda_c^+} \cdot \vec{n}_p) = -\cos\vartheta$ and the angular distribution becomes equal to

$$4\pi \frac{dB}{d\Omega_{\vec{n}_p}}(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{(\mathrm{L})} = 0.87\,(1 + 0.09\,\cos\vartheta). \tag{4.2}$$

Since the coefficient in front of $\cos\vartheta$ is rather small, so that the angular distribution of the probability of the decays is practically isotropic. Therefore, one can conclude that in the four–body mode $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ the charmed baryon Λ_c^+ seems to be practically unpolarized.

5 Conclusion

We have considered the four–body mode of the weak non–leptonic decay of the charmed Λ_c^+ baryon: $\Lambda_c^+ \to p + K^+ + \pi^+ + \pi^0$. Experimentally this is the most favourable mode among the four–body modes of the Λ_c^+ decays. From theoretical point of view this mode is rather difficult for the calculation, since baryonic and mesonic degrees of freedom are not fully factorized. However, as has been shown in Ref.[2] this problem has been overcome for the three–body mode $\Lambda_c^+ \to p + K^- + \pi^+$ within the effective quark model with chiral $U(3) \times U(3)$ symmetry incorporating Heavy Quark Effective Theory (HQET) and the ENJL model [2].

Following [2] we have calculated in the chiral limit the probability and angular distribution of the probability of the mode $\Lambda_c^+ \to p + K^+ + \pi^+ + \pi^0$ in the rest frame of the Λ_c^+ baryon and relative to the momentum of the daughter proton. The probability of the mode $\Lambda_c^+ \to p + K^+ + \pi^+ + \pi^0$ is obtained with respect to the probability of the mode $\Lambda_c^+ \to p + K^+ + \pi^+$. The theoretical prediction $B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+) = 0.87$ fits well the experimental data $B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{\mathrm{exp}} = (0.68 \pm 0.27)$. We would like to accentuate that in our approach the probability $B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)$ does not contain free parameters. Hence, such an agreement with experimental data testifies a correct description of low–energy dynamics of strong interactions in our approach.

The theoretical angular distribution of the probability of the decay mode $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ predicts a rather weak polarization of the charmed baryon Λ_c^+. This means that for the experimental analysis of the polarization properties of the Λ_c^+ produced in reactions of photo and hadroproduction the three–body decay mode $\Lambda_c^+ \to p + K^- + \pi^+$

seems to be preferable with respect to the four–body $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$. Nevertheless, the theoretical analysis of polarization properties of the charmed baryon Λ_c^+ in the weak non–leptonic four–body modes like (1) $\Lambda_c^+ \to \Lambda + \pi^+ + \pi^+ + \pi^-$, (2) $\Lambda_c^+ \to \Sigma^0 + \pi^+ + \pi^+ + \pi^-$ and (3) $\Lambda_c^+ \to p + \bar{K}^0 + \pi^+ + \pi^-$ with branching ratios [13]

$$\begin{aligned} B(\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-)_{\rm exp} &= (3.3 \pm 1.0)\,\%, \\ B(\Lambda_c^+ \to \Sigma^0\pi^+\pi^+\pi^-)_{\rm exp} &= (1.1 \pm 0.4)\,\%, \\ B(\Lambda_c^+ \to p\bar{K}^0\pi^+\pi^-)_{\rm exp} &= (2.6 \pm 0.7)\,\% \end{aligned}$$

comeasurable with the branching ratio of the mode $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ is rather actual and would be carried out in our forthcoming publications.

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